# Warm-Up

## CST: Algebra 2, 7.0/A.APR.7

$\frac{x^2+4x}{x+3} \bullet \frac{x^2-9}{x^2+x-12} =$

A. 1

B. $x$

C. $x+4$

D. $\frac{x+3}{x-3}$

## Review: Algebra 1, 13.0/A.APR.7

$\frac{5x^3}{10x^7} =$

A. $2x^4$

B. $\frac{1}{2x^4}$

C. $\frac{1}{5x^4}$

D. $\frac{x^4}{5}$

- How might a student obtain each of the incorrect answers?

## Current: Algebra 1, 13.0/A.APR.7

Which fraction equals the product $\left(\frac{x+5}{3x+2}\right)\left(\frac{2x-3}{x-5}\right)$?

A. $\frac{2x-3}{3x+2}$

B. $\frac{3x+2}{4x-3}$

C. $\frac{x^2-25}{6x^2-5x-6}$

D. $\frac{2x^2+7x-15}{3x^2-13x-10}$

## Other: Grade 7, NS 2.2

Julia bought $\frac{7}{8}$ of a pound of candy. Jen bought $\frac{1}{4}$ of a pound of candy. Which expression below represents the *sum* of their candy purchases?

A. $\frac{7}{8}\left(\frac{1}{4}\right)$

B. $\frac{7}{8}+\frac{1}{4}$

C. $\frac{7}{8}-\frac{1}{4}$

D. $\frac{7}{8}\div\frac{1}{4}$

- Which of the answer choices represents the difference in the total candy purchases?

**Today's Objectives/Standards:** (CA/CCSS) Algebra 1: 13.0/ A.APR.7 Algebra 2: 7.0/ A.APR.7
Students understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division; students add, subtract, multiply and divide rational expressions.

## Connecting Fractions and Rational Expressions: A Look Through the Grades

What "math" word do you see in "rational?" [Ratio]

What does that word mean to you? [Elicit student responses and include a few examples.]

**Definition**: A ratio is a comparison of numbers by division, typically expressed as $\frac{a}{b}$, where $b \neq 0$.

(There are several ways to express ratios; when working with rational expressions in Algebra and beyond, we typically see them as quotients.)

**Definition:** A rational number is any number that may be expressed as $\frac{a}{b}$, where *a* and *b* are contained in the set of integers and $b \neq 0$.

- When we do work with fractions, we are working with ratios. Thinking about how we work with fractions will assist us in thinking about and working with rational expressions.

**Opening for discussion:**
What types of operations are we asked to do with fractions? How do you know what to do? When? Why?

- Have students generate different examples of problems and tell you how to do them.
- Allow for students to list/explain multiple methods.
- Note that the list shown below is not an exhaustive list of methods but simply a sample of responses.

| Visual (example) | Operation | How to approach: |
|---|---|---|
| $\frac{8}{12}$ | **Simplify**<br>(Numerator and Denominator must be relatively prime, only common factor is 1) | Prime Factor and find equivalent forms of one.<br>Divide out common factors. |
| $\frac{7}{12} \bullet \frac{4}{21}$ *or* $\frac{3}{4}\left(\frac{8}{8}\right)$ | **Multiplication** | Multiply across, prime factor and find equivalent forms of one. |
| $\frac{-18}{25} \div \frac{36}{45}$ *or* $\frac{\frac{-1}{2}}{\frac{5}{10}}$ | **Division** | Divide across.<br>Find Common Denominators.<br>Multiply by the reciprocal. |
| $\frac{-11}{14} + \frac{6}{7}$ | **Addition** | Find common denominators and add numerators, use the Bubble Method. |
| $\frac{3}{5} - \frac{13}{7}$ | **Subtraction** | Find common denominators and subtract numerators, use the Bubble Method. |
| $\frac{10}{12} = \frac{x}{4}$ | **Proportion** | Multiply by the LCM.<br>Clear Fractions.<br>Cross-Product. |

**Note**: Keep this list in front of you! The thinking described in our list will help you decide how to approach each of the upcoming problems, even though they appear much more difficult.
**Note:** Factors are items that are multiplied; may appear as a numeric factor, variable factor or quantity factor.

**Example 1:** Simplify.

- Prompt the students: What do your instincts say to do? Should we factor? Are the numerators and denominators relatively prime? How do we know?

| Pre-Algebra | Algebra 1 (Easy) | Algebra 1 (Difficult) | Algebra 2 |
|---|---|---|---|
| $\frac{-32}{48}$<br><br>$=\frac{-1\bullet 2\bullet 2\bullet 2\bullet 2\bullet 2}{2\bullet 2\bullet 2\bullet 2\bullet 3}$<br><br>$=\frac{-1\bullet 2}{3}$<br><br>$=\frac{-2}{3}$ | $\frac{9xy^2}{36x^3y}$<br><br>$=\frac{3\bullet 3\bullet x\bullet y\bullet y}{2\bullet 2\bullet 3\bullet 3\bullet x\bullet x\bullet x\bullet y}$<br><br>$=\frac{y}{2\bullet 2\bullet x\bullet x}$<br><br>$=\frac{y}{4x^2}$ | $\frac{x^2+10x+24}{x^2+7x+6}$<br><br>$=\frac{(x+6)(x+4)}{(x+6)(x+1)}$<br><br>$=\frac{x+4}{x+1}$ | $\frac{2x^3+12x^2+10x}{12x+60}$<br><br>$=\frac{2x(x^2+6x+5)}{12(x+5)}$<br><br>$=\frac{2\bullet x\bullet (x+5)(x+1)}{2\bullet 2\bullet 3\bullet (x+5)}$<br><br>$=\frac{x(x+1)}{6}$ |

**You try!** Simplify. Classify each problem as being either "Algebra 1" or "Algebra 2" in terms of difficulty. Explain why you think one problem is considered more complex than the other.

| | |
|---|---|
| $\frac{x^2-13x+30}{x^2-12x+27}$<br><br>$=\frac{(x-10)(x-3)}{(x-9)(x-3)}$<br><br>$=\frac{x-10}{x-9}$ | $\frac{x^2-196}{(x-14)^2(x^2+16x+28)}$<br><br>$=\frac{(x-14)(x+14)}{(x-14)(x-14)(x+14)(x+2)}$<br><br>$=\frac{1}{(x-14)(x+2)}$ |

**Example 2:** Simplify. (Multiply.)

- Prompt students: What operation do we see here? How do we know to proceed despite the presence of negatives or variables? How do we know our thinking is correct?

| Pre-Algebra | Algebra 1 | Algebra 2 |
|---|---|---|
| $\frac{-5}{6} \bullet \frac{-18}{25}$ | $\frac{(x+1)}{3xyz} \bullet \frac{-15z}{x}$ | $\frac{3x^2+7x+2}{(x+2)} \bullet \frac{x^{-5}}{x(3x+1)}$ |
| $= \frac{-1 \bullet 5}{2 \bullet 3} \bullet \frac{-1 \bullet 2 \bullet 3 \bullet 3}{5 \bullet 5}$ | $= \frac{(x+1) \bullet -1 \bullet 3 \bullet 5 \bullet z}{3 \bullet x \bullet y \bullet z \bullet x}$ | $= \frac{(3x+1)(x+2)}{(x+2)} \bullet \frac{x \bullet x^{-6}}{x(3x+1)}$ |
| $= \frac{-1 \bullet 5 \bullet -1 \bullet 2 \bullet 3 \bullet 3}{2 \bullet 3 \bullet 5 \bullet 5}$ | $= \frac{-5(x+1)}{x^2 y}$ | $= \frac{(3x+1)(x+2) \bullet x \bullet x^{-6}}{(x+2) \bullet x(3x+1)}$ |
| $= \frac{1 \bullet 3}{5}$ | | $= x^{-6}$ |
| $= \frac{3}{5}$ | | $= \frac{1}{x^6}$ |

**Example 3:** Simplify. (Divide.)

- Prompt students: What operation do we see here? How do we know to proceed despite the presence of negatives or variables? How do we know our thinking is correct?

| Pre-Algebra | Algebra 1 | Algebra 2 |
|---|---|---|
| $\frac{3}{10} \div \frac{1}{5}$ | $\frac{9x^3}{44} \div -\frac{3x}{11}$ | $\frac{x^2+10x+24}{x^2-36} \div \frac{x^2+8x+16}{6x-36}$ |
| $= \frac{3}{10} \div \frac{1}{5} \bullet \frac{2}{2}$ | $= \frac{\left(9x^3\right) \div (-3x)}{44 \div 11}$ | $= \frac{x^2+10x+24}{x^2-36} \bullet \frac{6x-36}{x^2+8x+16}$ |
| $= \frac{3}{10} \div \frac{2}{10}$ | $= \frac{-3x^2}{4}$ | $= \frac{(x+4)(x+6)}{(x+6)(x-6)} \bullet \frac{6(x-6)}{(x+4)(x+4)}$ |
| $= \frac{3 \div 2}{10 \div 10}$ | | $= \frac{(x+4)(x+6) \bullet 6(x-6)}{(x+6)(x-6)(x+4)(x+4)}$ |
| $= \frac{3 \div 2}{1}$ | | $= \frac{6}{x+4}$ |
| $= \frac{3}{2}$ | | |

**You Try!** Simplify. Multiply or divide. (Use Sage/Scribe for this example.)

| | |
|---|---|
| $\frac{7x^5}{x^2+3x+2} \div \frac{7x^3}{x+2}$ | $\frac{5x^2+6x+1}{x^{-12}} \bullet \frac{x^8}{35x+7}$ |
| $= \frac{7x^5}{(x+2)(x+1)} \div \frac{7x^3}{(x+2)}$ | $= \frac{(5x+1)(x+1)}{x^{-12}} \bullet \frac{x^8}{7(5x+1)}$ |
| $= \frac{\left(7x^5\right) \div \left(7x^3\right)}{(x+2)(x+1) \div (x+2)}$ | $= \frac{(5x+1)(x+1) \bullet x^8 \bullet x^{12}}{7(5x+1)}$ |
| $= \frac{x^2}{x+1}$ | $= \frac{x^{20}(x+1)}{7}$ |

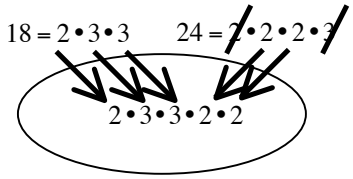
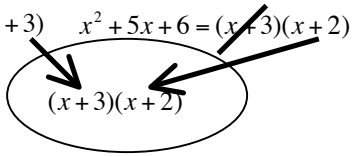
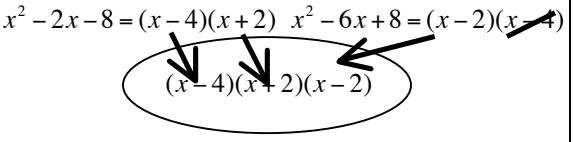
**Example 4:** Simplify. (Add the fractions.)

- Prompt students: What operation do you see? What do we need in order to add these fractions? How will we find it and create common denominators? (Turn and talk to your partner, justify!)

| Pre- Algebra | Algebra 1 | Algebra 2 |
|---|---|---|
| $20 = 2 \bullet 2 \bullet 5 \qquad 15 = 3 \bullet 5$<br>$2 \bullet 2 \bullet 5 \bullet 3$ | $x^2 = x \bullet x \qquad 2x = 2 \bullet x$<br>$x \bullet x \bullet 2$ | $x^2 - 15x + 44 = (x-11)(x-4) \qquad x^2 - 16 = (x+4)(x-4)$<br>$(x-11)(x-4)(x+4)$ |
| $\frac{3}{20} + \frac{4}{15} \qquad LCD = 60$ | $\frac{12}{x^2} + \frac{5}{2x} \qquad LCD = 2x^2$ | $\frac{6}{x^2-15x+44} + \frac{x}{x^2-16} \qquad LCD = (x-11)(x-4)(x+4)$ |
| $= \frac{3}{20} \bullet \frac{3}{3} + \frac{4}{15} \bullet \frac{4}{4}$ | $= \frac{12}{x^2} \bullet \frac{2}{2} + \frac{5}{2x} \bullet \frac{x}{x}$ | $= \frac{6}{(x-11)(x-4)} \bullet \frac{(x+4)}{(x+4)} + \frac{x}{(x-4)(x+4)} \bullet \frac{(x-11)}{(x-11)}$ |
| $= \frac{9}{60} + \frac{16}{60}$ | $= \frac{24}{2x^2} + \frac{5x}{2x^2}$ | $= \frac{6(x+4)}{(x-11)(x-4)(x+4)} + \frac{x(x-11)}{(x-11)(x-4)(x+4)}$ |
| $= \frac{9+16}{60}$ | | |
| $= \frac{25}{60}$ | $= \frac{24+5x}{2x^2}$ | $= \frac{6x+24}{(x-11)(x-4)(x+4)} + \frac{x^2-11x}{(x-11)(x-4)(x+4)}$ |
| $= \frac{5 \bullet 5}{2 \bullet 2 \bullet 3 \bullet 5}$ | | $= \frac{6x+24+x^2-11x}{(x-11)(x-4)(x+4)}$ |
| $= \frac{5}{12}$ | | $= \frac{x^2-5x+24}{(x-11)(x-4)(x+4)}$ |

**Example 5:** Simplify. (Subtract the fractions.)

- Prompt students: What operation do you see? What do we need in order to subtract these fractions? How will we find it and create common denominators? (Turn and talk to your partner, justify!)

| Pre-Algebra | Algebra 1 | Algebra 2 |
|---|---|---|
|   |   |   |
| $\frac{2}{18}-\frac{7}{24}$ $LCD=72$<br><br>$=\frac{2}{18}\bullet\frac{4}{4}-\frac{7}{24}\bullet\frac{3}{3}$<br><br>$=\frac{8}{72}-\frac{21}{72}$<br><br>$=\frac{-13}{72}$ | $\frac{5}{x+3}-\frac{2}{x^2+5x+6}$ $LCD=(x+3)(x+2)$<br><br>$=\frac{5}{(x+3)}\bullet\frac{(x+2)}{(x+2)}-\frac{2}{(x+2)(x+3)}\bullet\frac{1}{1}$<br><br>$=\frac{5(x+2)}{(x+3)(x+2)}-\frac{2}{(x+3)(x+2)}$<br><br>$=\frac{5x+10}{(x+3)(x+2)}-\frac{2}{(x+3)(x+2)}$<br><br>$=\frac{5x+10-2}{(x+3)(x+2)}$<br><br>$=\frac{5x+8}{(x+3)(x+2)}$ | $LCD=(x-4)(x+2)(x-2)$<br><br>$\frac{x-2}{x^2-2x-8}-\frac{x-1}{x^2-6x+8}$<br><br>$=\frac{x-2}{(x-4)(x+2)}-\frac{x-1}{(x-2)(x-4)}$<br><br>$=\frac{(x-2)}{(x-4)(x+2)}\bullet\frac{(x-2)}{(x-2)}-\frac{(x-1)}{(x-2)(x-4)}\bullet\frac{(x+2)}{(x+2)}$<br><br>$=\frac{(x-2)(x-2)}{(x-4)(x+2)(x-2)}-\frac{(x-1)(x+2)}{(x-4)(x+2)(x-2)}$<br><br>$=\frac{x^2-4x+4}{(x-4)(x+2)(x-2)}-\frac{x^2+x-2}{(x-4)(x+2)(x-2)}$<br><br>$=\frac{x^2-4x+4-[x^2+x-2]}{(x-4)(x+2)(x-2)}$<br><br>$=\frac{x^2-4x+4-x^2-x+2}{(x-4)(x+2)(x-2)}$<br><br>$=\frac{-5x+6}{(x-4)(x+2)(x-2)}$ |

**You Try!** Simplify.

- Have students work in pairs. Play "Pass the Pen" to debrief.

| | |
|---|---|
| $\frac{4x}{x+3}-\frac{4x}{x+6}$ | $\frac{2}{3x^2+12x}+\frac{8}{2x}$ |
| $=\frac{4x(x+6)}{(x+3)(x+6)}-\frac{4x(x+3)}{(x+6)(x+3)}$ | $=\frac{2}{3x(x+4)}+\frac{8}{2\bullet x}$ |
| $=\frac{4x^2+24x}{(x+3)(x+6)}-\frac{4x^2+12x}{(x+3)(x+6)}$ | $=\frac{2}{3x(x+4)}\bullet\frac{2}{2}+\frac{8}{2\bullet x}\bullet\frac{3(x+4)}{3(x+4)}$ |
| $=\frac{4x^2+24x-4x^2-12x}{(x+3)(x+6)}$ | $=\frac{4}{6x(x+4)}+\frac{24(x+4)}{6x(x+4)}$ |
| $=\frac{12x}{(x+3)(x+6)}$ | $=\frac{4+24x+96}{6x(x+4)}$ |
| | $=\frac{24x+100}{6x(x+4)}$ |
| | $=\frac{2\bullet 2\bullet(6x+25)}{2\bullet 3\bullet x(x+4)}$ |
| | $=\frac{2(6x+25)}{3x(x+4)}$ |

**Exit Ticket**

Select two problems below. Using the Sage/Scribe approach, complete both problems. For one problem, Partner A writes, Partner B talks. For the other problem, switch roles. Turn the work into your teacher. Be sure to use proper syntax! (Post problems and do a gallery walk; display solutions to each problem.)

1. $\frac{7}{3}-\frac{8}{12x-8}$

2. $\frac{5}{n+5}+\frac{4n}{2n+6}$

3. $\frac{x+6}{x^2+5x-6}$

4. $\frac{a-4}{a^2-2a-8}\div\frac{1}{a-5}$

5. $\frac{x+7}{7x+35}\bullet\frac{x^2-3x-40}{x-8}$